# Analysis of influencing factors of mortar projectile reproduction process on fragment mass distribution 

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#### Abstract

: This paper deals with high explosive projectile reproduction process over several years of monitoring and testing its fragmentation characteristics. Experimental data used in analysis were obtained using PIT tests in our country. Many fragmentation tests were conducted with 19 different production series of projectile mortar projectile 120 mm M62, in a time span of over three years. Number, mass and fragments shape of each fragment mass group are determined using the PIT test. In PIT test, warhead is detonated in closed space, filled with sand. After the fragmentation of warhead, fragments are obtained from the sand. Mass and shape of fragments are determined, and fragments are classified by their mass groups. Number of methods is available for prediction of fragment mass distribution, and one of these is method introduced by M. Held. Authors tried to find variations in reproduction process of projectile, using statistical parameters of given tests. In order to analyze possible significant differences between tests, Student $t$-test was introduced.


Keywords: reproduction process; mortar projectile; statistical analysis;

## 1 Introduction

After the initial detonation of explosive in naturally fragmenting projectiles, body of projectile randomly splits into large number of fragments. Shape, mass, number and spatial distribution of natural fragments depends on projectile body geometry (internal and external path), mechanical properties of case materials (tensile strength, yield strength, toughness, and thermal processing) as well as characteristics of explosive (density and detonation pressure and velocity). Even though mortar projectiles are two-dimensional axi-symmetric bodies, fragment spatial distribution around detonating projectile is not uniform, so prediction of fragments spatial density parameters is complex problem. Number of fragments, their mass, geometrical shapes and spatial distribution are determined experimentally, with Pit and Arena test method.

In this research authors performed thorough analysis of large number of fragmentation tests performed in Pit and Arena facilities for mortar projectile 120 mm . Aim of analysis was to establish correlation between individual tests in particular testing year, as well for all tests together, and also to find possible differences in tests, since tests were conducted in longer time period, from 1986. to 1989. year. Using time period as a variable proved adequate since interesting results came out of the research. Authors declassified all available experimental data from fragmentation tests conducted in Bosnia and Herzegovina for this type of projectile.

## 2 Experimental plan

More than 40 experimental fragmentation tests in Pit and Arena were carried out with mortar projectile 120 mm M62. Technical data for tested mortar projectile are given in table 1.

In Pit test, projectiles were detonated in closed space, filled with sand of particular granulation. After detonation, fragments were collected from the sand. Method of collecting fragments can make significant influence on overall test results.

By reducing man made errors (approach with magnet collector) results in Pit fragmentation test that are usually more accurate. Once the fragments were collected, their mass and number determined, they were accordingly classified into particular fragment groups depending on their mass.

Table 1: Technical data for mortar projectile 120 mm M62 [11]

| Technical data |  |
| :--- | :--- |
| Total mass | 12600 g |
| Mass of explosive (TNT) | 2250 g |
| Case material | Steel 9189 VP (JAS) <br> impact, <br> Fuze |
|  | super quick, and <br> delayed action |
| Total mass | 12600 g |

Spatial distribution of fragments was determined using data from Arena with four sectors. Each sector represented one quarter of circle and this type of Arena was usual in our country before semicircular Arena was introduced. In Arena fragmentation test, mortar projectile was put in vertical, nose-down, position at a certain distance from the ground. After detonation, number of hits and perforations in Arena wooden sectors were counted and then hits number and perforations number per square meter were calculated for each sector.

## 3 Methods used in research

For prediction of parameters of natural fragmentation process and overall analysis of results, authors used experimental researches, and analytical, numerical and statistical methods. Lately there have been attempts to use CAD modeling approach in conjunction with numerical methods [10].

### 3.1 Fragment velocities

In real battlefield scenario, projectile fragment velocity is resultant of initial fragment velocity, translation component of projectile velocity and rotational velocity of projectile (if there is rotation imparted to projectile). When considering mortar projectile dynamics, usually there is no axial rotation, so initial fragment velocity can be approximated using Gurney's formula. Errors made using this approximation are significantly small. General expression of Gurney formula for cylindrical configuration is:

$$
\begin{equation*}
v_{\text {Gurney }}=\sqrt{2 \cdot E} \cdot \sqrt{\frac{1}{\left(0,5+\frac{M}{C}\right)}} \tag{1}
\end{equation*}
$$

where $\sqrt{2 \cdot E}$ is Gurney constant, $M$ - metal mass of projectile case and $C$ - mass of explosive charge [2]. The Gurney constant can be approximated using expression $\sqrt{2 \cdot E}=0,338 \cdot D$, where $D$ is detonation velocity, depending on explosive type and its density.

### 3.2 Radius of projectile efficiency

Dependency of fragments density per $\mathrm{m}^{2}$ as a function of distance can be established if there are available experimental data from Arena tests. Fragments concentration density per arena sector's area at certain distance from the projectile detonation center is given with formula:

$$
\begin{equation*}
d\left(R_{i}\right)=\frac{N_{p e n}}{A} \tag{2}
\end{equation*}
$$

where $N_{\text {pen }}$ is number of perforations at arena sector made of wooden panels, $A$ is the sector's area and $R$ is radial distance between the projectile explosion center and a certain sector.

Using approximation method it is easy to find best fitting curve for available experimental data. It is usually power function in general form $y=a \cdot x^{b}$.

Furthermore, a characteristic distance, at which fragments density is one effective perforation per square meter, can be obtained. For this analysis, interpolation technique is used. This distance is known as projectile efficiency radius and it is overall recognized that projectiles with greater efficiency radius have greater terminal efficiency.

### 3.3 Mass distribution of fragments

For prediction of fragment mass distribution, the Held formula with two parameters and the total mass $M_{0}$ gives an excellent description of the experimentally found mass distributions of a natural fragmented warhead (Held, 1993). A fit to natural-fragmentation data can be obtained using equation:

$$
\begin{equation*}
M(n)=M_{0} \cdot\left(1-e^{-B n^{2}}\right) \tag{3}
\end{equation*}
$$

where $B$ and $\lambda$ are both empirically determined constants with B of order $10^{-2}$ and $\lambda$ of order $2 / 3$ [8]. In the Held equation $M_{0}$ is the total mass of all fragments, $M(n)$ and $n$ are the cumulative fragments mass and cumulative fragments number beginning with the heaviest fragment.

Held frequently found that it was necessary to discard a few of the heaviest fragments in order to obtain a curve fit to data over the rest of the range. The constants B and $\lambda$ are determined from above equation by mathematical transformation:

$$
\begin{equation*}
\left[M_{0}-M(n)\right] / M_{0}=e^{-B n^{2}} \tag{4}
\end{equation*}
$$

and the natural logarithm of the above equation is:

$$
\begin{equation*}
\ln \left[\left(M_{0}-M(n)\right) / M_{0}\right]=-B \cdot n^{\lambda} \tag{5}
\end{equation*}
$$

If the logarithm of above equation is performed again, it is possible to determine the constants $B$ and in the log-log plot. By differentiating the equation (3), Held gave the approximate mass of the n -th fragment:

$$
\begin{equation*}
m(n)=d M(n) / d n=M_{0} \cdot B \cdot \lambda \cdot n^{\lambda-1} \cdot e^{-B \cdot n^{2}} \tag{6}
\end{equation*}
$$

From the fragment mass distribution log-log diagram, constant $B$ and exponent $\lambda$ with appropriate correlation coefficient $r^{2}$ can be obtained.

If in the log-log diagram, the straight line does not fit the measuring data very well, given total mass $M_{0}$ was not an optimum mass for such fragments mass distribution. Now, an optimum mass (or best mass) $M_{0 B e s t}$ is calculated using following equation:

$$
\begin{equation*}
M_{0_{\text {Bet }}}=M(n) /\left(1-e^{-B n^{2}}\right) \tag{7}
\end{equation*}
$$

The new constants $B_{B}$ and $\lambda_{B}$ are now determined using the total mass $M_{0 B e s t}$ :

$$
\begin{equation*}
M(n)=M_{0_{\text {Bet }}} \cdot\left(1-e^{-B_{B} \cdot n^{n_{B}}}\right) \tag{8}
\end{equation*}
$$

This procedure is repeated until a satisfactory correlation coefficient is obtained ( $r^{2} \geq 0,99$ )

Another approach in analysis of fragment mass distribution is representing the experimental data as dependence of fragments number on average mass of fragments in particular mass group, namely $N_{f r}$ vs $m_{\text {aver }}$. In this kind of analysis sometimes data are densely packed, particularly because of larger fragment mass groups, so it is usually appropriate to represent the data in the form of log-log plot.

### 3.4 Descriptive statistics

In the research authors used descriptive statistics as a method to describe the basic features of data obtained in experimental study. There were two major characteristics of a single variable that were analyzed: central tendency and dispersion.

The central tendency of a distribution generally is an estimate of the center of a distribution of values. The arithmetic mean of a set of $N$ numbers $X_{1}, X_{2}, X_{3}, \ldots X_{N}$ is denoted by $\bar{X}$ and is defined as [12]:

$$
\begin{equation*}
\bar{X}=\frac{X_{1}+X_{2}+X_{3}+\ldots+X_{N}}{N}=\frac{\sum_{i=1}^{N} X_{i}}{N}=\frac{\sum_{N} X}{N} \tag{9}
\end{equation*}
$$

The standard deviation of a set of $N$ numbers $X_{1}, X_{2}, X_{3}, \ldots X_{N}$ is denoted by $s$ and is defined by:

$$
\begin{equation*}
s=\sqrt{\frac{\sum_{i=1}^{N}\left(X_{i}-\bar{X}\right)^{2}}{N}}=\sqrt{\overline{(X-\bar{X})^{2}}} \tag{10}
\end{equation*}
$$

where $X$ represents the deviations of each of the numbers $X_{i}$ from the mean $\bar{X}$. Thus $s$ is the root square of the deviations from the mean, or, as it is sometimes called, the root-meansquare deviation [12].

### 3.5 Statistical hypotheses

Statistical hypotheses are generally statements about the probability distributions of the populations. Procedures that enable us to determine whether observed samples differ significantly from the results expected, and thus help us decide whether to accept or reject hypotheses, are called tests of hypotheses, tests of significance, and rules of decision [12].

In testing a given hypothesis, the maximum probability with which we would be willing to risk is called the level of significance of the test. In practice, a significance level of 0,05 or 0,01 is customary, although other values are also used.

If, for example, the 0,05 (or $5 \%$ ) significance level is chosen in designing a decision rule, then there are about 5 chances in 100 that we would reject the hypothesis when it should be accepted; that is, we are about $95 \%$ confident that we have made the right decision. In such case we say that the hypothesis has been rejected at the 0,05 significance level, which means that the hypothesis has a 0,05 probability of being wrong [12].

For analysis of differences between results of experimental tests, Student $t$ test was introduced. The purpose of using Student $t$ test was to assess whether the means of two groups are statistically different from each other.

Suppose that two random samples of sized $N_{1}$ and $N_{2}$ are drawn from normal populations whose standard deviations are equal, and that these two samples have means given by $\overline{X_{1}}$ and $\overline{X_{2}}$, as well as standard deviations given by $s_{1}$ and $s_{2}$, respectively. To test the hypothesis $H_{0}$ that the samples come from the same population, $t$ score (or t statistic) is used, and it is usually given by [12]:

$$
\begin{equation*}
t=\frac{\overline{X_{1}}-\overline{X_{2}}}{s_{\overline{x_{1}}-\overline{x_{2}}}} \tag{11}
\end{equation*}
$$

where $s_{\overline{x_{1}}-\overline{x_{2}}}$ is standard error of the difference. To compute it, one must use formula:

$$
\begin{equation*}
s_{\overline{x_{1}}-\overline{x_{2}}}=s \cdot \sqrt{\frac{1}{N_{1}}+\frac{1}{N_{2}}} \tag{12}
\end{equation*}
$$

where standard deviation $s$ is determined by following expression:

$$
\begin{equation*}
s^{2}=\frac{\left(N_{1}-1\right) \cdot s_{1}^{2}+\left(N_{2}-1\right) \cdot s_{2}^{2}}{N_{1}+N_{2}-2} \tag{13}
\end{equation*}
$$

The $t$ value will be positive if the first mean value is larger than the second and negative if it is smaller. Degrees of freedom ( $d f$ ) for the test also need to be determined. In the $t$ test, the degrees of freedom are the sum of the samples in both groups minus value 2 (eq. 13). Given the significance level, degrees of freedom, and $t$ value, next step to look the $t_{\text {critical }}$ value in a standard table of significance (available as an appendix in most statistics books) to determine whether the $t$ value is large enough to be significant. If it is, conclusion arises that the difference between the means for the two groups is significant.

## 4 Analysis and discussion of results

Since the sophisticated measuring equipment for fragment velocity measurement was not available to authors, the Gurney's formula was used in order to predict initial fragments velocities. Methodology used was described in Crull's report [2].

Combination of analytical and CAD methods was used in order to complete this task, since 3D model of mortar projectile 120 mm M62 was split into large number of quasi-cylindrical segments along its symmetry axis, and Gurney formula was applied for every segment. Even though this can be time consuming process it does gives us opportunity to inspect and analyze valuable data for fragment initial velocities.

From diagram of initial fragment velocities vs. relative distance of projectile case segments (fig 2.) it can be seen that mean fragment velocity for mortar projectile 120 mm M62 is around $1100 \mathrm{~m} / \mathrm{s}$. Since fragment initial velocities, according to Gurney method, depend on ratio of projectile case mass to explosive charge mass, it is obvious that larger initial velocities can be noticed in segments where this ratio is greater.

Range of velocities varied between $600 \mathrm{~m} / \mathrm{s}$ for fragments originating from lower part of projectile, where large fragments occur, and $1200 \mathrm{~m} / \mathrm{s}$ for fragments coming from front and cylindrical part of projectile body. Higher values of initial velocities of fragments are obtained from mortar projectiles since stresses on its body during the projectile motion through the barrel are not as high as with artillery projectile. This means that, combined with higher impact angles and lower values of $M / C$ ratio gives mortar projectile better terminal efficiency.


Figure 1: Initial velocity of fragments for mortar projectile 120 mm M62
In analysis of mortar projectile 120 mm M62 radius of efficiency, authors performed analysis by using combined approximation-interpolation technique, based on experimental data from Arena tests. Fragments density per $\mathrm{m}^{2}$ as a function of distance was determined using equation (2). Approximation of test data and interpolation, in order to determine projectile radius of efficiency, was performed.

Figure 2. shows power approximation curve of averaged experimental data for fragment density (frag $/ \mathrm{m}^{2}$ ) vs. distance from (m). Data on diagram in fig. 2 are presented in form of stack, with min. and max. value of fragment density, and average values in the middle. Characteristic distance, at which fragments density is one effective perforation per $\mathrm{m}^{2}$, was obtained and this distance represents the radius of efficiency. Radius of efficiency for mortar projectile 120 mm M62 was 16,15 meters. That means that in a circle with radius of $16,15 \mathrm{~m}$ there is high probability that a


Figure 2: Radius of efficiency determination for mortar projectile 120 mm M62 soldier will be incapacitated.

Initial analysis of mass distribution of fragments was performed using widely accepted Held methodology. In order to get better agreement with experimental data, Held's constants $B$ and $\lambda$ were obtained using iteration procedure for obtaining optimum mass $M_{\text {obest }}$ (eq. 7). This was necessary since for some tests, given total mass $M_{0}$ of collected fragments was not an optimum mass for corresponding fragments mass distribution. Values of constants $B$ and $\lambda$ are presented in table 2.

From diagrams for different testing years (figure 4) it is obvious that Held's prediction formula shows overall good agreement with test data. Small deviation of Held's prediction curves from experimental data occurs for smaller mass groups. This is often the case since smaller mass groups are more sensitive to differences in the process of experimental testing and man-made errors.


Figure 3: Mass distribution of fragments for each year (1986. - 1989.) using Held's method
In order to try to better interpret Held's prediction formula, Held's constants $B$ and $\lambda$ were analyzed and average values are presented in table 2. Expected values of constants $B$ were of order 0,01 and $\lambda$ of order 0,75 [8].

From analysis performed, authors found average values of constants $B$ and $\lambda$ to be of expected order [8]. Best agreement with expected values of constants was obtained for tests conducted in year 1989.

Table 2: Values of Held's constant $B$ and $\lambda$ for different years of testing

|  | $\mathbf{B}_{\text {aver }}$ year | $\lambda_{\text {aver/jear }}$ |
| :---: | :---: | :---: |
| $\mathbf{1 9 8 6}$ year | 0,045 | 0,650 |
| $\mathbf{1 9 8 7}$ year | 0,031 | 0,620 |
| $\mathbf{1 9 8 8}$ year | 0,031 | 0,660 |
| $\mathbf{1 9 8 9}$ year | 0,010 | 0,781 |
| Average values | 0,029 | 0,677 |

By looking at the results from table 2, highest value of constant B is determined for 1986. year and lowest for year 1989. From experimental data results, lowest number of fragments, particularly in mass groups smaller than 1 g , were reported for year 1986., and highest for 1989. This may lead to important conclusion that constant $B$ is extremely sensitive to changes (more than constant $\lambda$ ) in fragment number, especially for lower fragment groups. Collection of fragments from experimental tests from 1986. were carried out without electro-magnet so obtained results were prone to man-made errors during the process of manipulation with fragments. This can have significant influence on fragment mass distribution.

Another approach in analysis of experimental data was finding the relation between fragments number and average mass of fragments in particular mass group, $N_{f r}$ vs $m_{\text {aver }}$, where data were presented in log-log plot (fig. 4).

From fig. 4 it can be seen that for fragment mass groups smaller than 20 g , higher number of fragments were recorded in 1989, which is consistent with our previous claims that tests in
1989. show higher number of fragments $N$ and total mass of collected fragments $m_{t o t}$ in these mass groups, comparing to the results of other testing years.

Interesting part of diagram (fig. 4) is relation of experimental data for years 1987. and 1988. These data matches significantly, and looking at the average values of Held's constant $B$ for these two years, one can notice that it is the same - 0,031 .


Figure 4: Diagram of $N_{f r}$ vs $m_{\text {aver }}$, log-log scale
Even though previous analysis can give valuable insight into complexity of problem, authors developed new method for representing experimental data. In figure 5. diagram is shown where experimental data was presented in form $N_{i} / N$ and $m_{i} / m$ vs. average mass of fragments $m_{\text {aver }}(g)$ in particular mass group, where $N_{i}$ is cumulative fragments number starting from lowest mass group, $N$ is total number of collected fragments, $m_{i}$ is cumulative fragments mass starting also from lowest mass group and $m$ is total mass of collected fragments. Diagram has two ordinates. Plot presented in fig 5. is convenient for representing large amount of data since data overlapping is avoided.

When analyzing $N_{i} / N$ vs $m_{\text {aver }}(g)$ curves (left ordinate in fig. 5) for different years of testing, variation for tested years were not significantly different, comparing to $m_{i} / m$ vs $m_{\text {aver }}(g)$ diagrams. It is important to notice, similar as in diagram in fig. 4, that ratio $N_{i} / N$ is largest for fragments collected in 1989., and lowest for fragments from year 1986. Using this representation it is easier to compare results and derive certain conclusions, particularly because second ordinate presents $m_{i} / m$ vs $m_{\text {aver }}(g)$ curves.

In $m_{i} / m$ vs $m_{\text {aver }}(g)$ diagrams (right ordinate in fig. 5), significant variation in diagrams can be noticed. Higher fragments number $N_{i}$ and fragments mass $m_{i}$ proportions relative to total number of fragments $N$ and total mass of fragments $m$ were present in year 1989., and lowest values were recorded for year 1986. It is obvious that high variation is present among all testing years when analyzing masses of fragments.

Plots in fig. 5 only confirms earlier authors suggestions that higher number of fragments $N$ collected in year 1986., strongly influence mass distribution of fragments, and analysis of constant $B$ can point to differences in experimental data.

It can now be concluded that dominant influencing factors on Held's constant $B$ value, and overall mass distribution of fragments, are: way of conducting the experiment, types of fragment collecting methods, man-made errors during the experiment, variations in thermal processing of materials during production, oscillations in methods of forging (numerical or standard machines), differences in characteristics of materials. From a study [13] on variations of explo-
sive charge density in axial and radial directions in projectile, results show that differences in explosive density can vary up to $5 \%$ for TNT explosive.


Figure 5: Method proposed by authors for analysis of ratio of number of fragments \& total number of fragments $N_{i} / N$, and mass of fragments \& total mass of fragments $m_{i} / m$

On of the focuses of this research was using statistical tools, such as Descriptive statistics and Student $t$ test to determine possible variations between experimental fragmentation test.

Diagrams in figures 6 and 7 show summary relative standard deviations results for experimental data sets from 1986. to 1988. First diagram represents relative standard deviation of fragments number $N_{\text {std.dev }} / N_{\text {aver }}(\%)$ in particular mass groups, and second one shows standard deviation of fragments mass $m_{\text {std.dev }} / m_{\text {aver }}(\%)$ in certain mass groups, for all testing years.

Highest standard deviations, regarding fragments number $N$, were recorded for year 1988. (for mass groups $>1 \mathrm{~g}$ ) and overall lowest standard deviations for year 1986. (fig. 6). Larger mass groups ( $>30 \mathrm{~g}$ ) have significant deviations in number of fragments.


Figure 6: Relative standard deviations of fragment number $N_{\text {std.dev }} / N_{\text {aver }}$ vs. particular mass group, for different years of testing (1986. - 1988.)


Figure 7: Relative standard deviations of fragments average mass $m_{\text {std.der }} / m_{\text {aver }}$ vs. particular mass group, for different years of testing (1986. - 1988.)
When analyzing bar diagram in fig. 7, highest standard deviations can be reported for mass group smaller than 0,5 g., particularly for years 1987. and 1988. Largest single deviation is reported for mass group 4-5 g in year 1987 (around 11\%).

Diagrams in fig. 6. and 7. present valuable data, since they are the result of over 40 fragmentation Pit tests, occurred during time span of 3 years.

Analysis of possible differences among different tests was performed using Student $t$ test. The purpose of using $t$ test was to assess whether the means $(\bar{X})$ of two data groups are statistically different from each other. Authors pursued two approaches.

In the first approach, using $t$ test, null hypothesis was set up:

$$
\begin{equation*}
\mathrm{H}_{0}: \overline{X_{i}}=\bar{X}_{\text {aver }}, \tag{14}
\end{equation*}
$$

and accordingly alternative hypothesis:

$$
\begin{equation*}
\mathrm{H}_{1}: \bar{X}_{i} \neq \bar{X}_{\text {aver }} . \tag{15}
\end{equation*}
$$

where $\bar{X}_{i}$ was average value of data in individual test, and $\bar{X}_{\text {aver }}$ average value of all data in certain year of testing. This means that every individual fragmentation test was compared to average values of all test results in particular year of testing, in order to find whether significant differences exist. For given tests, if obtained $t$ value (eq. 11) was significantly large (larger than $t_{\text {critical }}$ value in a standard statistics table, for chosen level of significance), it can be concluded that there is a significant difference among testing data.

Authors chose level of significance of $\alpha=0,05$, since that is common practice in most researches. That means that if a null hypothesis was true ( $t<t_{\text {critical }}=2,0555$ ), with $95 \%$ level of confidence it can be concluded that there is no significant difference in tested data sets. Authors performed two-tailed independent two-sample $t$ test, assuming equal sample sizes and variance. Analysis was done using test with 26 degrees of freedom (14 samples in each test class).

Values of Student test $t$ parameter (eq. 9) for fragments number $N$ and fragment mass $m(g)$ were calculated for every hypothesis test, and results for all testing years are presented in fig. 8 and 9 . Abscise in diagrams represent individual test that is statistically compared to average values for given testing year.

When using first approach in analysis, authors found that maximal value of $t$ for fragments number $N$ for all tested years was 0,95 (1987. year). Variations of other values of $t$ are presented in fig 8 . Overall highest values of statistical parameters $t$ were found for year 1987. As can be seen from diagram in fig. 9, values of $t$ for fragment masses $m$ were significantly smaller, compared with $t$ values for fragment number $N$. Analysis shows that, since the tabular value of $t_{\text {critical }}(2,0555)$ was much larger (minimum 2 times) than calculated $t$ values for all tests (fragment number $N$ and mass $m$ ), authors concluded with $95 \%$ level of confidence that there was no significant difference in data sets.


Figure 8: Values of Student test $t$ parameter for fragments number $N$


Figure 9: Values of Student test $t$ parameter for fragments mass $m$ In second approach null hypothesis was:

$$
\begin{equation*}
\mathrm{H}_{0}: \overline{X_{i}}=\bar{X}_{j}, \tag{16}
\end{equation*}
$$

and accordingly alternative hypothesis:
$\mathrm{H}_{1}: \bar{X}_{i} \neq \bar{X}_{j}$.

In equation (17) $\overline{X_{i}}$ was average value of data in individual test $i$, and $\bar{X}_{j}$ average value of data in individual test $j$ for particular year. Second approach was actually comparation of possible significant differences between all tests individually, for all testing years separately. This was a daunting task since authors performed more than 700 individual statistical hypothesis testing, where every test was compared to every other test in certain testing year.

Procedure was much like in the first approach, with level of significance of $\alpha=0,05, t_{\text {critical }}$ $=2,0555$ and 26 degrees of freedom. Authors, similar as in previous case, performed twotailed independent two-sample $t$ test, assuming equal sample sizes and variance.

Table 3 presents results (min. and max. values of calculated value $t$ ) obtained using second approach where comparation of possible significant differences between all tests individually, and for all testing years, was performed.

Table 3: Values of statistical parameter $t$ for fragment number $N_{f r}$ and fragments mass $m_{f r}(g)$

|  | $\mathbf{N}_{\text {fr }}$ |  | $\mathbf{m}_{\text {fr }}(\mathbf{g})$ |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $\mathbf{t}_{\text {min }}$ | $\mathbf{t}_{\max }$ | $\mathbf{t}_{\min }$ | $\mathbf{t}_{\max }$ |
| $\mathbf{1 9 8 6}$ | 0,00749 | 0,85935 | 0,00071 | 0,14224 |
| $\mathbf{1 9 8 7}$ | 0,00799 | 1,43853 | 0,00102 | 0,15626 |
| $\mathbf{1 9 8 8}$ | 0,10002 | 1,01907 | 0,01381 | 0,06957 |

For fragments number $N_{f r}$ maximum value of $t$ was 1,43 for year 1987, and for fragment mass $m_{f r}(g)$ maximum value $t$ was 0,156 , also for year 1987. Trend of $t$ values calculated was similar as in the first approach. All $t$ values (table 3) were much smaller than tabular value of two-tailed $t$ critical $(2,0555)$, especially for fragment masses $m_{f r}(g)$ class. All this means that with $95 \%$ level of confidence it can be said that there was no significant difference in individual data sets, and for all testing years.

## 5 Conclusions

Reproduction process over several years of monitoring and testing its fragmentation characteristics was analyzed. More than 40 experimental fragmentation tests in Pit and Arena were carried out with mortar projectile 120 mm M62.

Initial velocity of fragments ( $600-1200 \mathrm{~m} / \mathrm{s}$ ) and radius of efficiency $(16,15 \mathrm{~m})$ for projectile 120 mm were determined using Gurney formula.

Held's constants $B$ and $\lambda$ were analyzed and authors found average values of constants $B$ and $\lambda$ to be of expected order. Authors conclude that constant $B$ is extremely sensitive to changes (more than constant $\lambda$ ) in fragments number $N$, especially for lower fragment groups $(<1 \mathrm{~g})$. This can occur due to mishandling during test manipulation and types of fragments collecting methods.

Besides Held's formula, $N_{f r}$ vs $m_{\text {aver }}$ approach was used, which was particularly suited for analysis when applied $\log -\log$ scale plot. By comparing results for constants $B$ and $\lambda$, and diagram $N_{f r}$ vs $m_{\text {aver }}$, authors confirmed previous conclusion that higher number and mass of fragments could correspond with lower values of constant $B$.

Authors introduced new method, where experimental data were presented in form $N_{i} / N$ and $m_{i} / m$ vs. average mass of fragments in particular mass group. Dominant influencing factors on Held's constant $B$ value are: methods of conducting the experiment, fragment collecting ways, man-made errors during the experiment, variations in thermal processing of materials for projectile body, oscillations in process of forging, differences in applied production technologies.

Descriptive statistics were used in order to determine possible variations between fragmentation test. Larger mass groups have significant deviations in number of fragments. Regarding
fragment masses $m_{i}$ highest standard deviations can be reported for mass group smaller than 0,5 g., particularly for years 1987. and 1988.

Analysis of possible differences among different tests was performed using Student $t$ test. Two approaches were used - every individual fragmentation test was compared to average values of all tests in particular year of testing, and comparation of possible significant differences between all tests individually for all testing years separately. Analysis shows value of $t_{\text {critical }}$ to be much larger than calculated $t$ values for all tests (for fragment number $N$ and fragment mass $m$ ), so authors concluded with $95 \%$ level of confidence that there was no significant difference in data sets.

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